Vibration and Damping Analysis of a Multilayered Cylindrical Shell, Part II: Numerical Results

Naiyar Alam*
Aligarh Muslim University, Aligarh, India
and
N. T. Asnani†
Indian Institute of Technology, New Delhi, India

The solution for the vibration and damping analysis of a general multilayered cylindrical shell consisting of an arbitrary number of orthotropic material elastic and viscoelastic layers with simply supported end conditions has been reported in Part I of this paper. The solution has been programmed on an IBM 360 computer for evaluating the resonant frequency parameters and the associated system loss factors for all families of modes having nonaxisymmetric and axisymmetric vibrations of the general multilayered shell. The reasonating frequency parameters and associated system loss factors and their variations with geometrical and material parameters for three-, five-, and seven-layered shells with alternate elastic and viscoelastic layers are reported herein.

Introduction

IN Part I of this paper,[1] the governing equations of motion for nonaxisymmetric and axisymmetric vibrations of a general n-layered shell consisting of an arbitrary number of orthotropic material layers have been derived by variational principles. Included in the analysis are extension, bending inplane shear, and transverse shear deformations in all of the layers and circumferential, longitudinal translatory, and transverse inertias. The solution for a radially, simply supported shell has been evaluated by taking the series solution and procedure for evaluating the resonating frequencies and the associated system loss factors for all $(2n+3)$ modes of vibration reported.[1]

A computer program for determining the resonant frequencies and associated loss factors for nonaxisymmetric and axisymmetric vibrations for $(2n+3)$ modes of an n-layered shell with alternate elastic and viscoelastic layers has been developed. The number of layers and their geometrical and material parameters are fed as input data and the frequency parameters and system loss factors are the output results obtained. In the program, the elements of matrices $[\hat{A}_1]$ and $[B_1]$ for the nonaxisymmetric vibrations and the elements of $[\hat{A}_2]$, $[B_2]$, $[\hat{A}_3]$, and $[B_3]$ for the axisymmetric vibrations given in Part I are determined. The problem is transformed to the standard form of an eigenvalue problem and all the complex eigenvalues are evaluated. The real part of the complex eigenvalue gives the resonant frequency parameter and the ratio of the imaginary part to the real part of the complex eigenvalue gives the associated system loss factor. Thus, the vibration and damping analysis of a general multilayered shell of an arbitrary number of elastic and viscoelastic layers of orthotropic materials may be obtained by using this program. Resonating frequency parameters and associated system loss factors for all families of the modes of vibration for three-, five-, and seven-layered shells with elastic and viscoelastic layers have been evaluated and their variations with geometric and material property parameters have been developed, keeping the total thickness of the layered shells the same. The results presented here provide useful data for the proper choice of shell parameters to obtain the maximum possible system loss factors for different modes of vibrations.

Identification of Modes

The various modes of vibration of a multilayered shell are identified on the basis of displacements in the modes. The longitudinal displacements $u_1, u_2, \ldots$ and circumferential displacements $v_1, v_2, \ldots$ are expressed as ratios of transversal displacement w. Depending on the relative values of these ratios and their sign, i.e., positive or negative, the nature of modes is decided.

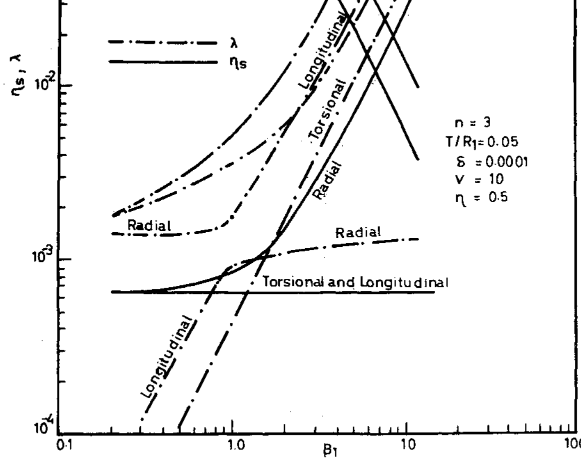

Fig. 1 Variation of λ and η_s with β_1 for axisymmetric vibrations of three-layered cylindrical shell.

Received Jan 28, 1983; revision received March 4, 1983.

*Reader, Mechanical Engineering Department.

†Associate Professor, Mechanical Engineering Department.

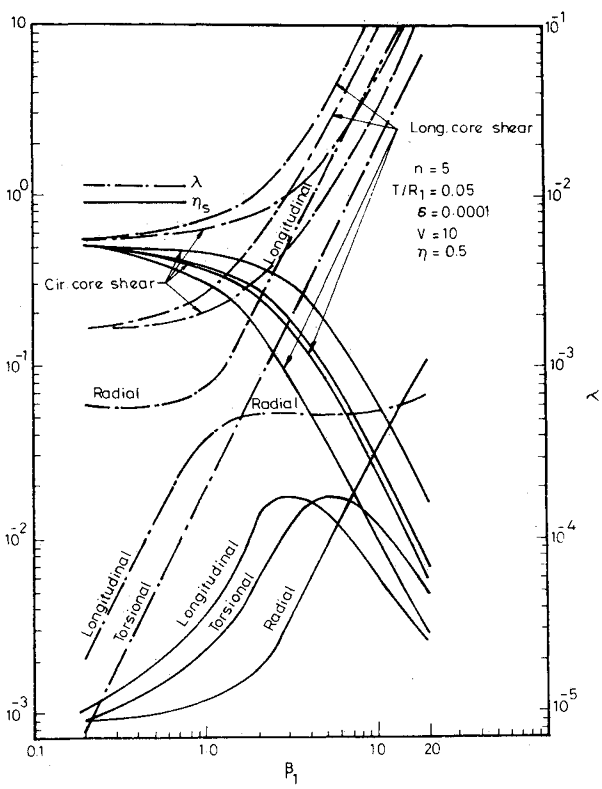

Fig. 2 Variation of λ and η_s with β_l for axisymmetric vibrations of five-layered cylindrical shell.

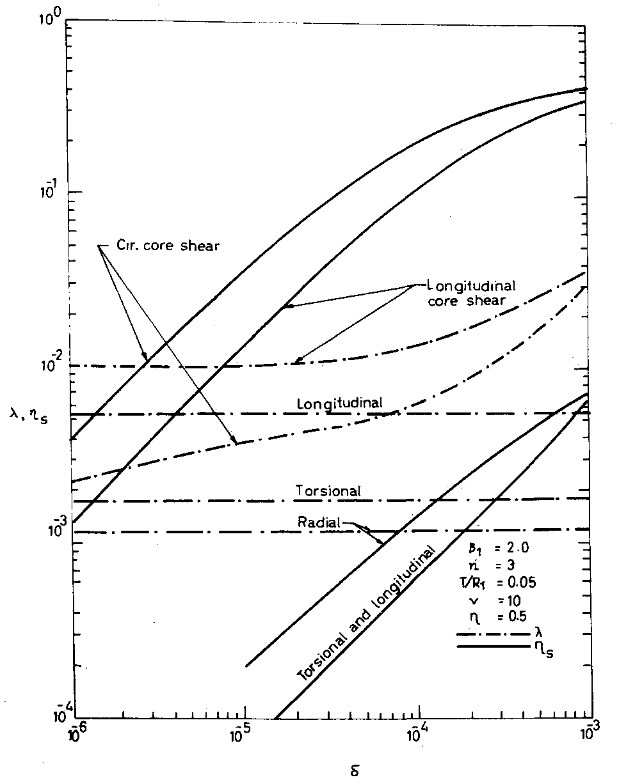

Fig. 4 Variation of λ and η_s with δ for axisymmetric vibrations of three-layered cylindrical shell.

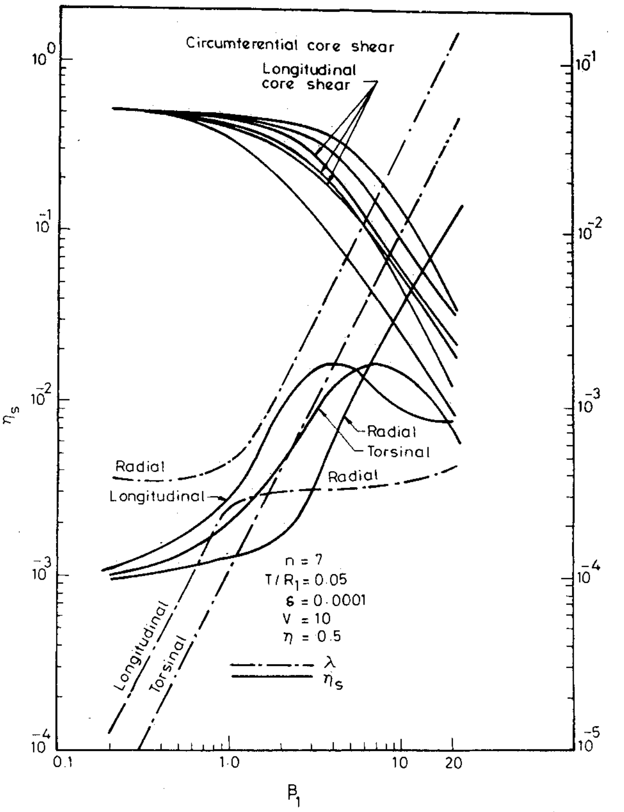

Fig. 3 Variation of λ and η_s with β_l for axisymmetric vibrations of seven-layered cylindrical shell.

For nonaxisymmetric vibrations of an n-layered shell, there are $(2n+3)$ coupled modes that are named according to their predominant displacements. Mode I, in which radial displacements are large relative to other displacements, is identified by small values of all the displacement ratios. The modes with predominant extensional and torsional displacements are recognized by the large values of same signs in the displacement ratios and are designated modes II and III. Modes IV, V, VI, etc., $(n-1)$ in number, are mainly due to the transverse shear of the soft core layers in the longitudinal and circumferential directions and are identified by the opposite direction displacements of one or more pairs of stiff layers. The remaining $(n+1)$ modes are designated as mainly stiff layer thickness shear modes.

For axisymmetric vibrations in an n-layered shell, there are two families of modes, a first family of $(n+2)$ modes having coupled radial and longitudinal displacements and a second of $(n+1)$ modes having only circumferential displacements. There is no coupling among the modes of the two families, as is evident from the two sets of uncoupled matrix equations.[1] Mode I of the first family is a radial mode identified by small values of displacement ratios and mode II is a longitudinal one identified by nearly the same large longitudinal displacement ratios of the same sign. The remaining n modes are due mainly to the shear of the soft cores and stiff layers along the longitudinal direction identified by some $+ve$ and some $-ve$ longitudinal displacements. In the second family there is one torsional mode identified by circumferential displacements of the same sign and n shear modes due to the shear of the soft cores and stiff layers in the circumferential directions identified by some $+ve$ and some $-ve$ circumferential displacements.

Multilayered Shell

The multilayered shell considered in the present study consists of alternate elastic and viscoelastic layers such that the face layers are always elastic and as such n is always an odd number. All of the elastic layers are assumed to be of the same thickness and isotropic material, as are all of the viscoelastic layers. The ratio of the thickness of the viscoelastic layer to that of the elastic layer is denoted by v. The ratio of the in-phase component of the shear modulus of the viscoelastic material to the Young's modulus of the elastic material is denoted by δ. Poisson's ratio of the elastic material ν is taken to be 0.3, and the ratio of the Poisson's ratio of the

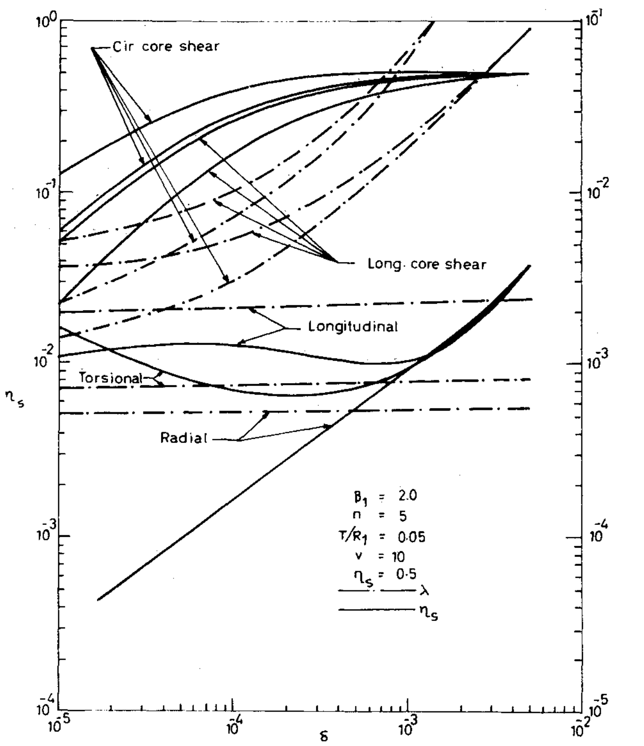

Fig. 5 Variation of λ and η_s with δ for axisymmetric vibrations of five-layered cylindrical shell.

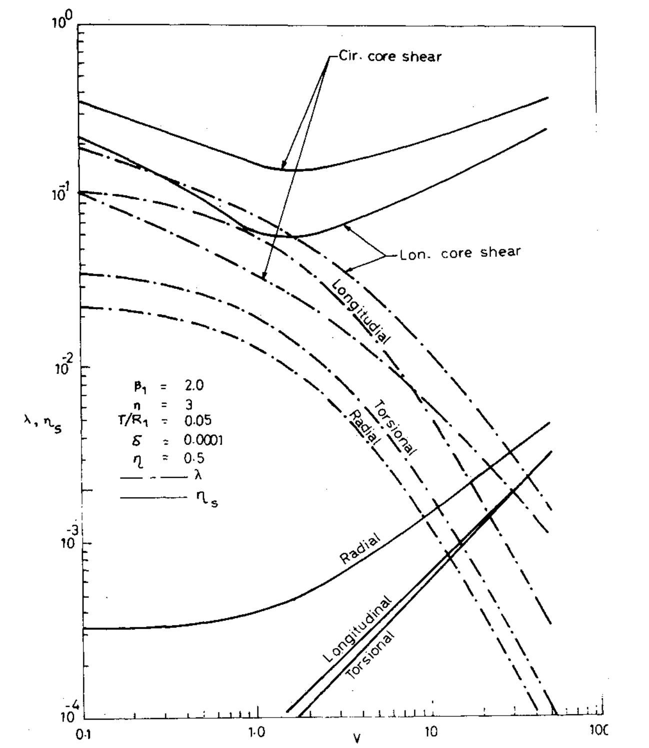

Fig. 7 Variation of λ and η_s with v for axisymmetric vibrations of three-layered cylindrical shell.

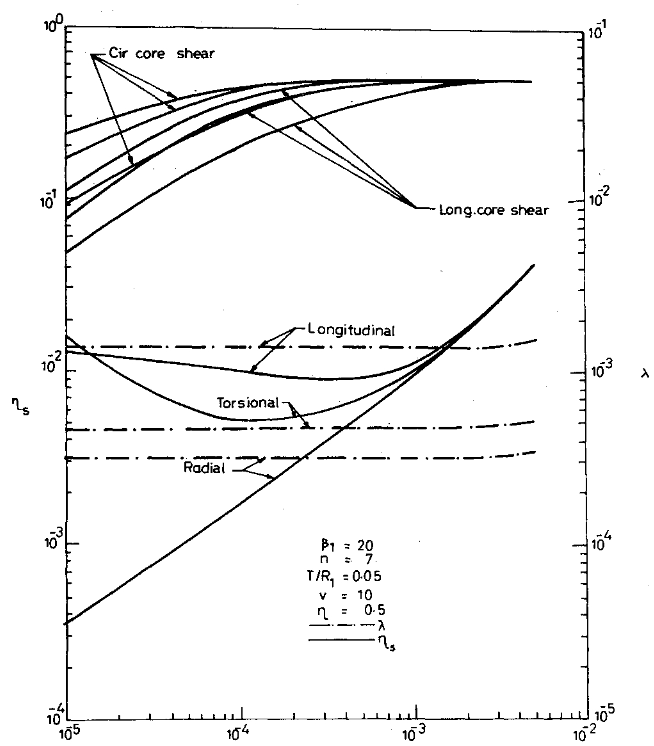

Fig. 6 Variation of λ and η_s with δ for axisymmetric vibrations of seven-layered cylindrical shell.

viscoelastic material to that of the elastic material Ψ is taken to be 1.33. The ratio of the mass density of the viscoelastic material to that of the elastic material γ is taken to be 0.5. The loss factor η of the viscoelastic material in shear as well as in extension is taken to be 0.5. Other parameters are 1) $\beta_l = m\pi R_l/L$, where m is the modal number specifying the half-sine wave modes along the axial direction, R_l the inner radius, and L the length of the shell; 2) T/R_l, where T is total thickness of the multilayered shell; 3) J is the modal number denoting half-sine wave modes along the circumference for nonaxisymmetric vibrations; and 4) n is the number of layers in the shell.

Results and Discussion

The results of a vibration and damping analysis of three-, five-, and seven-layered shells are presented in the form of frequency parameter $\lambda = (\rho t R_l \omega^2)/E$ and system loss factor η_s. Here ρ, t, and E are the mass density, thickness, and Young's modulus of the elastic layer, respectively, R_l the inner radius of the shell, and ω the resonating frequency in radians/s. The parameter T/R_l has been kept the same for all of the layered shells in order to compare their performance with constant size. The variation of λ and η_s with the nondimensional parameters β_l, δ, and v for axisymmetric and nonaxisymmetric vibrations are given in following sections for all modes except the stiff layer shear modes which are expected to possess negligible damping. λ for only the first three modes is plotted for the seven-layered shells to avoid overcrowding in the figures.

Variation of λ and η_s with β_l for Axisymmetric Vibration

Figures 1-3 show the variation in λ and η_s with β_l for the three-, five-, and seven-layered shells with $T/R_l = 0.05$, $\delta = 10^{-4}$, and $v = 10$. For all of the shells, λ remains nearly constant for lower values of β_l, decreases at $\beta_l = 1$, and again increases slightly with a further increase in β_l. For the radial mode, η_s increases with β_l for all shells, which means that it is higher for shorter shells and higher frequencies. For this mode, η_s increases with the number of layers in the shell for a particular value of β_l.

For the longitudinal and torsional modes, λ increases with β_l for all three shells. For these modes η_s is small for a three-layered shell and does not change with β_l, but there is a considerable increase in η_s in these modes when the number of layers is increased. For the five- and seven-layered arrangements, η_s increases when β_l reaches a certain value

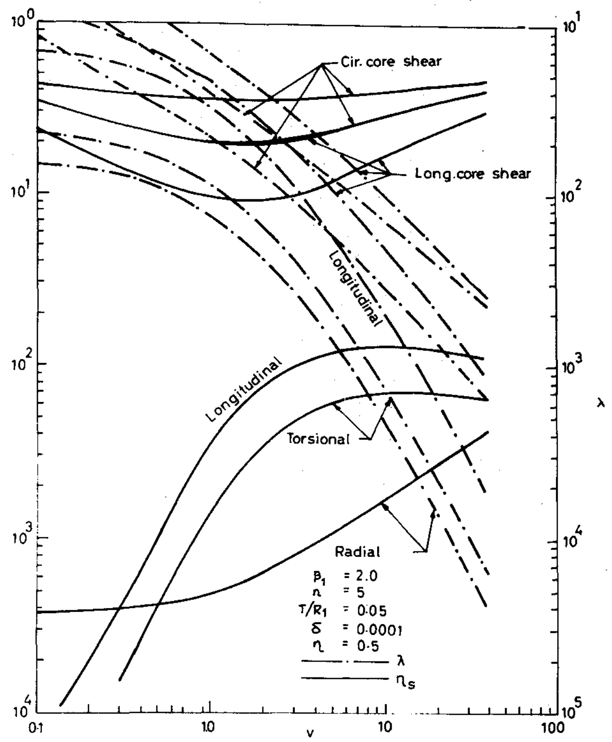

Fig. 8 Variation of λ and η_s with v for axisymmetric vibrations of five-layered cylindrical shell.

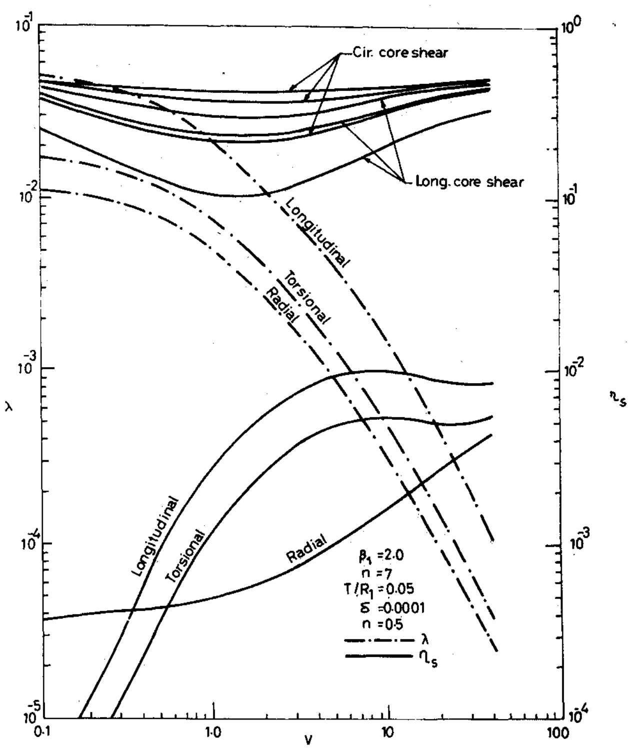

Fig. 9 Variation of λ and η_s with v for axisymmetric vibrations of seven-layered cylindrical shell.

and decreases with further increases in β_l. For the longitudinal and torsional modes having particular values of β_l, η_s is maximized in the five-layered arrangement and is reduced marginally with further increases in the number of layers. The maximum values of η_s for the longitudinal and torsional modes are nearly equal, but occur at different values of β_l.

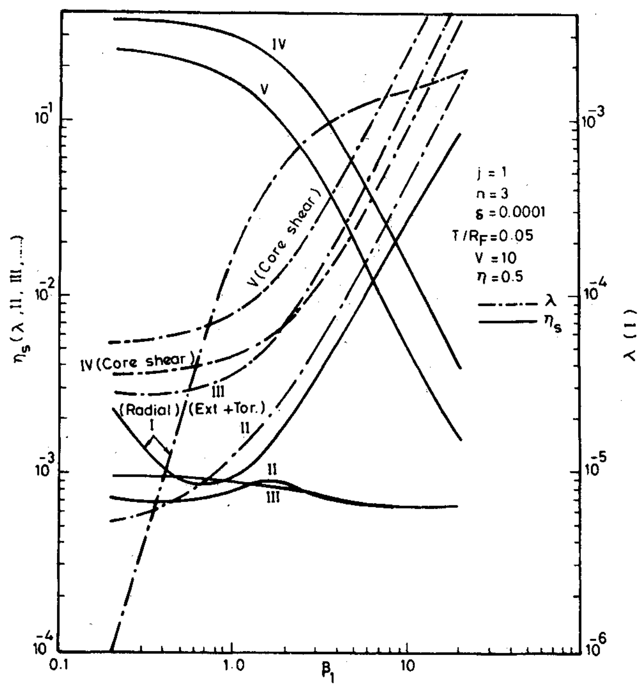

Fig. 10 Variation of λ and η_s with β_l for nonaxisymmetric vibrations of three-layered cylindrical shell.

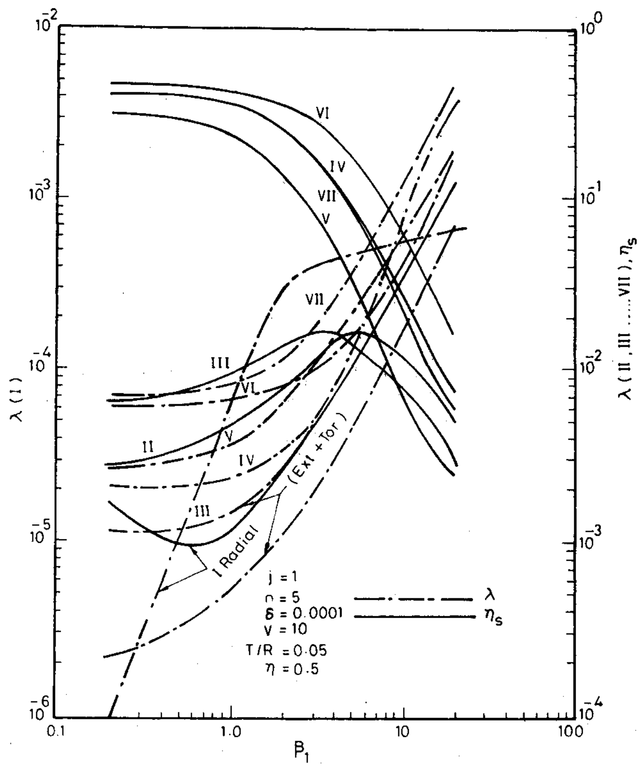

Fig. 11 Variation of λ and η_s with β_l for nonaxisymmetric vibrations of five-layered cylindrical shell.

For longitudinal and circumferential core-shear modes, λ increases with β_l for multilayered shells. For these modes of the three-, five-, and seven-layered shells at a low value of $\beta_l = 0.2$, η_s is equal to the material loss factor of the core and reduces with increasing values of β_l. For these modes η_s increases with the number of layers for a particular value of β_l; thus, a high value of η_s is available for a wider range of β_l with more layers.

A cylindrical shell with more layers gives reasonably high values of η_s for all families of modes with a proper selection

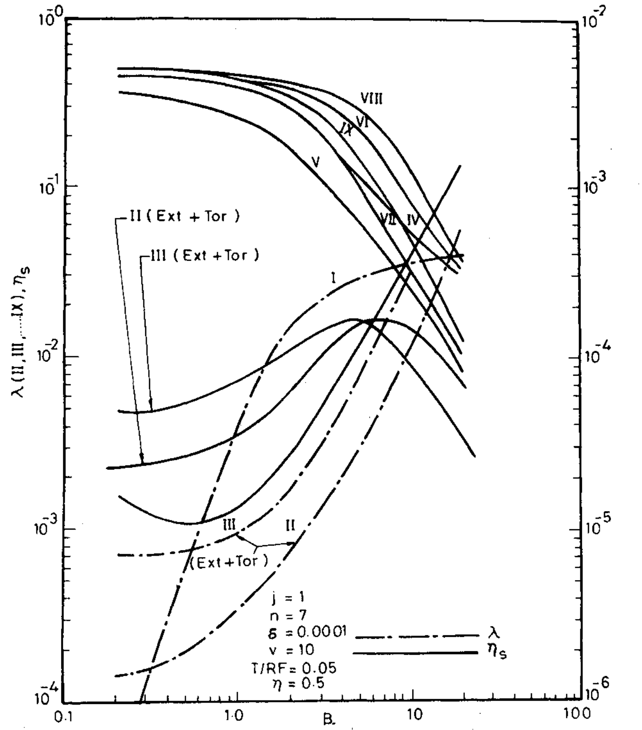

Fig. 12 Variation of λ and η_s with β_l for nonaxisymmetric vibrations of seven-layered cylindrical shell.

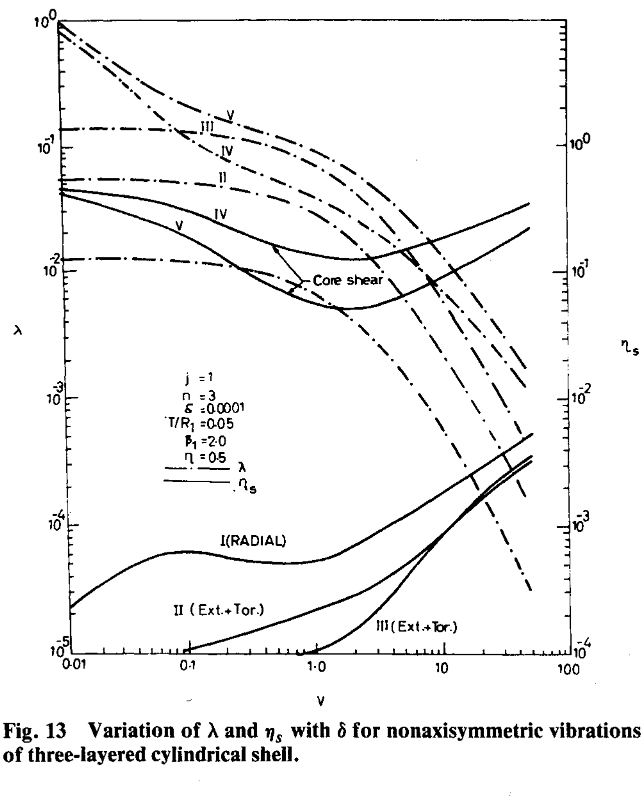

Fig. 13 Variation of λ and η_s with δ for nonaxisymmetric vibrations of three-layered cylindrical shell.

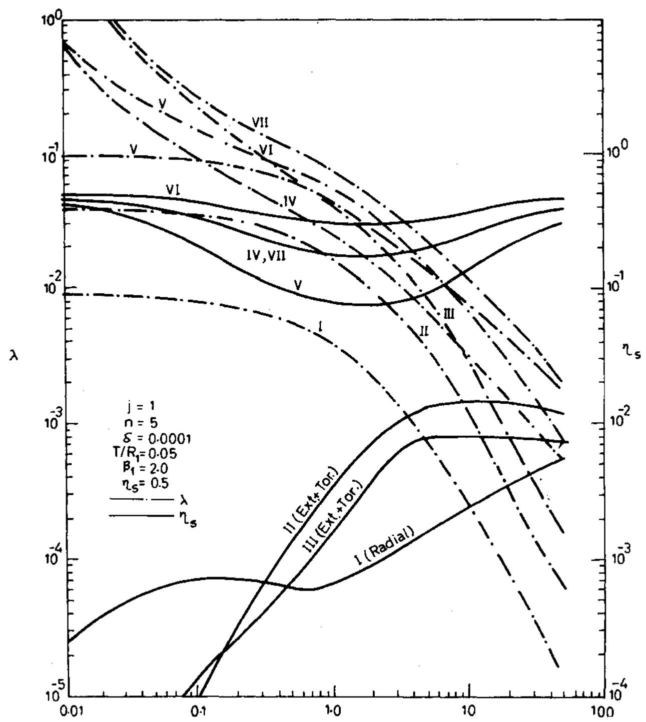

Fig. 14 Variation of λ and η_s with δ for nonaxisymmetric vibrations of five-layered cylindrical shell.

of β_l. For the typical example, $\beta_l = 5$ with a seven-layered shell gives reasonably high values of η_s for all families of modes.

Variation of λ and η_s with δ for Axisymmetric Vibrations

The variation of λ and η_s with δ for three-, five-, and seven-layered shells is shown in Figs. 4-6 for $\beta_l = 2$, $T/R_l = 0.05$, and $v \simeq 10$. For the radial mode λ shows no appreciable change in the chosen range of δ; only a marginal increase in λ is seen at higher values of δ with increased numbers of layers in the multilayered shell. For radial mode η_s increases with δ for all of the shells and there is only a marginal increase with increasing numbers of layers for a particular value of δ.

For the longitudinal and torsional modes, λ shows no appreciable change with δ. In fact, only a small increase in λ is observed at $\delta = 10^{-3}$ for the seven-layered shell. For a three-layered sandwich shell, for these modes, η_s is very small at lower values of δ, but there is a steep rise for higher values of δ. For these modes η_s is higher for the five-layered shell even with soft cores. There is a slight decrease in η_s in the longitudinal and torsional modes of the seven-layered shell.

For longitudinal and circumferential core shear modes, λ increases with δ. For these modes η_s increases with δ and also increases with the number of layers for a particular value of δ. Thus, to obtain a uniformly high value of η_s for all families of modes, one should choose a higher value of δ and more layers in the multilayered shell. In the present example, $\delta > 0.0005$ gives high values of η_s for all resonating modes with seven layers in the multilayered shell.

Variation of λ and η_s with v for Axisymmetric Vibrations

The variations of λ and η_s with v for axisymmetric vibration in the three-, five-, and seven-layered shells is shown in Figs. 7-9 for $T/R_l = 0.05$, $\beta_l = 2$, and $\delta = 10^{-4}$. For all modes λ decreases with the increasing value of v, as the stiffness of the shell reduces with increases in v. For the radial mode η_s increases with v, but there is only a marginal increase in η_s of this mode with additional layers. Thus, to obtain high values of η_s for the radial mode, one should go for higher values of v.

For the longitudinal and torsional modes, η_s is very low for a three-layered sandwich shell, especially for lower values of v, as compared with a five-layered shell. For these modes η_s reduces marginally in a seven-layered shell for a particular value of v. A similar trend is expected with further increases in the number of layers.

For the longitudinal and circumferential core shear modes, η_s is maximum either at very low values of v or at very high values of v. η_s is higher for these modes with more layers in

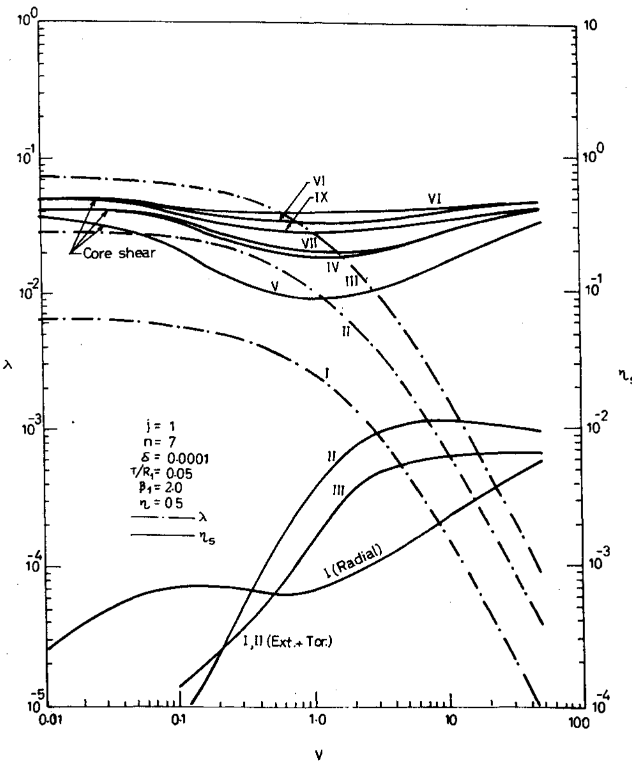

Fig. 15 Variation of λ and η_s with δ for nonaxisymmetric vibrations of seven-layered cylindrical shell.

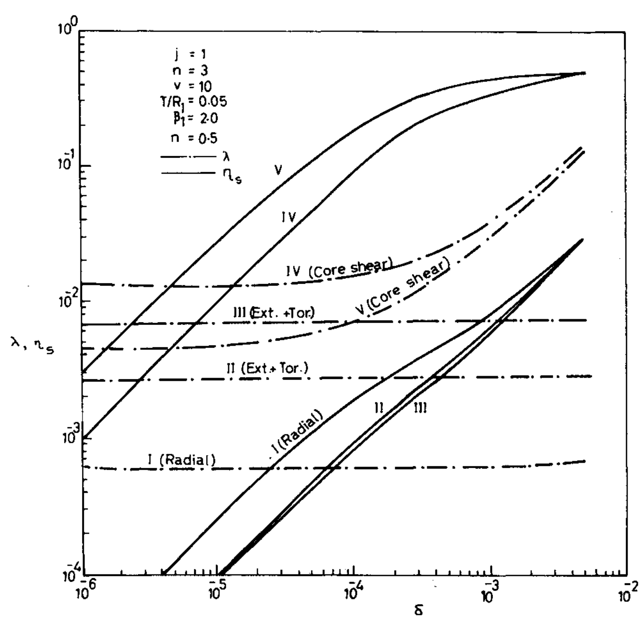

Fig. 16 Variation of λ and η_s with v for nonaxisymmetric vibrations of three-layered cylindrical shell.

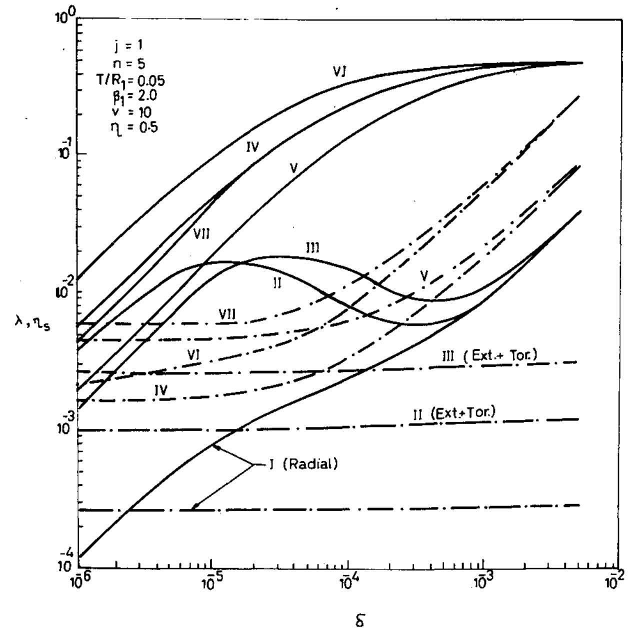

Fig. 17 Variation of λ and η_s with v for nonaxisymmetric vibrations of five-layered cylindrical shell.

the multilayered shell for a particular value of v. Thus, a uniformly high value of η_s for all families of modes is obtained for $v > 10$.

Variation of λ and η_s with β_l for Nonaxisymmetric Vibrations

Figures 10-12 show the variation of λ and η_s with β_l for $J = 1$, $T/R_l = 0.05$, $\delta = 10^{-4}$, and $v = 10$. For the radial mode λ increases with β_l for all three shells. For this mode η_s decreases with the increase in β_l, reaching a minimum and then increasing with further increases in β_l for three-, five-, and seven-layered shells. For the radial mode with values of β_l of 0.2-0.4, η_s decreases with the increasing number of layers, but for $\beta_l > 0.4$ it increases with the number of layers for a particular value of β_l.

For modes II and III (extension + torsion) λ increases with β_l for three-, five-, and seven-layered shells. For these modes η_s increases considerably when the number of layers in the shell are increased from three to five. For modes II and III, η_s increases with β_l reaching a maximum and then decreasing with further increases in β_l. For modes II and III the maximum values of η_s are nearly equal, but occur at different values of β_l.

Due to the thickness shear of the core layers in the axial and circumferential directions for higher-order modes λ increases with β_l for multilayered shells. For these modes η_s is at a maximum for very low values of β_l, decreasing with increasing values of β_l. η_s also increases with the number of layers in the shell, the curves becoming flatter with the increasing number of layers. Thus, a high value of η_s is available for those modes for a wider range of β_l with more layers in the shell.

A cylindrical shell with more layers gives reasonably high values of η_s for all families of modes with proper selection of β_l.

Variations of λ and η_s with δ for Nonaxisymmetric Vibrations

Variation of λ and η_s with δ for $J = 1$, for three-, five-, and seven-layered shells is shown in Figs. 13-15 for $T/R_l = 0.05$, $v = 10$, and $\beta_l = 2.0$. For the radial mode λ does not appreciably change for the chosen range of δ and only marginal increases are observed at higher values of δ. For radial mode η_s increases with δ for three-, five-, and seven-layered shells. There is an increase in η_s for this mode for a particular value of δ when the number of layers in the shell are increased from three to five, but only a marginal increase when the layers are further increased to seven.

For modes II and III there is no appreciable change in λ with δ and only a small increase is observed for higher values of δ. For these modes η_s is small for a three-layered shell in comparison to the corresponding values for a five-layered shell, especially at lower values of δ. For modes II and III maximum values of η_s are nearly equal for the five- and seven-layered shells, but they occur at different values of δ. A seven-layered shell seems to give higher values of η_s in these

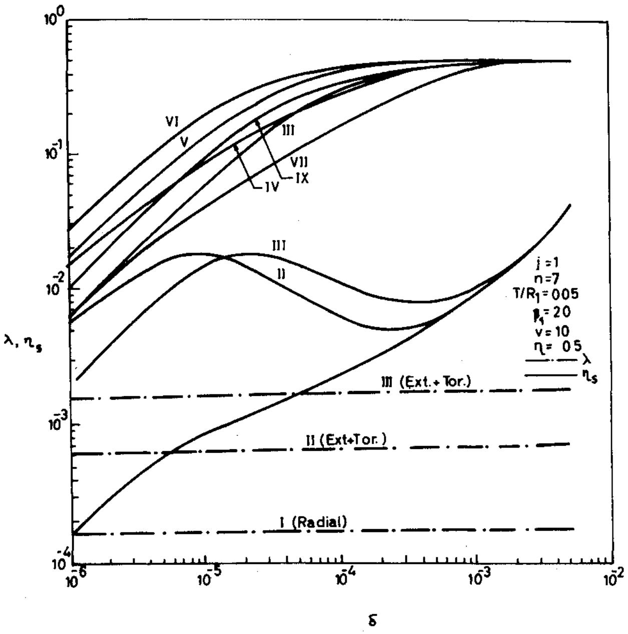

Fig. 18 Variation of λ and η_s with v for nonaxisymmetric vibrations of seven-layered cylindrical shell.

modes for a certain range of δ and the trend is reversed in some other ranges of δ. For modes I, II, and III, η_s is nearly the same for high values of δ in a multilayered shell.

For higher modes, due to thickness shear of cores, η_s increases with δ and also increases with the number of layers in the multilayered shell for a particular value of δ. Thus, to obtain a uniformly high value of η_s for all families of modes, one should choose higher values of δ and more layers in the multilayered shell.

Variation of λ and η_s with v for Nonaxisymmetric Vibrations

The variation of λ and η_s with v, for $J=1$, for three-, five-, and seven-layered shells is shown in Figs. 16-18 for $T/R_l=0.05$, $\delta=10^{-4}$, and $\beta_l=2.0$. For all modes of these shells, λ reduces with increasing values of v. For the radial mode η_s increases from $v=0.01$ to 0.1, decreases from $v=0.1$ to ≈ 1, and again increases for higher values of v. For the radial mode of a five-layered shell η_s is higher than for the three-layered shell for a particular value of v, but this increase is only marginal with a further increase in the number of layers to seven.

For modes II and III η_s is low for a three-layered shell especially for lower values of v, as compared with a five-layered shell. There is a marginal reduction in η_s for these modes when the number of layers in the shell is increased from five to seven.

For the core thickness shear modes η_s is maximum at either a very low or very high value of v and it is minimum at some intermediate value of v. For these modes η_s is higher with more layers in the multilayered shell for a particular value of v. Thus, we see that a uniformly high value of η_s for all families of modes is obtained for $v>10$.

Variations in λ and η_s with β_l, δ, and v for $J=2$ has been studied and it was found that these variations are of nearly the same nature as for $J=1$. It is expected that the same type of variations may also be accurate for higher values of J.

References

[1] Alam, N. and Asnani, N. T., "Vibration and Damping Analysis of a Multilayered Cylindrical Shell, Part 1: Theoretical Analysis," *AIAA Journal*, Vol. 22, June 1984, pp. 803-810.